Two models of imperfect delayed repair in a continuously monitored system and subject to a continuous deterioration

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AMMSI: **A**geing and **M**aintenance in reliability: **M**odelling and **S**tatistical **I**nference

Outline

Introduction

Condition-Based maintenance

- Condition-Based Maintenance (CBM) is a maintenance program that recommends to perform maintenance actions based on the information collected through condition monitoring
- CBM attempts to avoid unnecessary maintenance tasks by performing maintenance actions only when there is evidence of abnormal behaviour of the system
- A CBM program properly implemented can significantly reduce the total maintenance costs

Introduction

Imperfect maintenance

- Under a CBM program, based on the information data, different maintenance actions are programmed
- After a maintenance action, system condition depends on the maintenance efficiency. Two extreme cases
  - Minimal maintenance: system condition is just the same as before (ABAO)
  - Perfect maintenance: system condition is the same as if it were new (AGAN)
- Reality lies between these two extreme cases: Imperfect maintenance
- Imperfect maintenance has been widely investigated in the literature. However, its implementation in CBM is limited

Introduction

General framework

- A **deteriorating** system
- **Continuous monitoring**
- **Delay time** for the maintenance team arrival
- **Imperfect repair** performed by the maintenance team
- **Maintenance strategy** based on the system condition

Introduction

General Assumptions

- System subject to a continuous degradation and continuously monitored
- System failure, **maintenance team is called for repairing the broken system**
- The maintenance team **takes a fixed amount of time to start the repair** ("delayed repair") and the **system is unavailable**. This repair is instantaneous
- Maintenance strategy: **reduce the system downtime**. The maintenance team is called to perform a maintenance action before the system failure. It takes a fixed time to start the maintenance action
  - If the **system is failed at maintenance action time**: corrective replacement
  - If the **system is working at maintenance action time**: imperfect repair based on
    - System degradation reduction (First Maintenance Model)
    - System age reduction (Second Maintenance Model)

Formulation of the problem

General situation

- The degradation is modelled by a gamma process $(X_t)_{t\geq 0}$ where X_t is distributed $Gamma\left(\alpha t, \beta\right)$ with density

$$f_t\left(x\right) = \frac{\beta^{\alpha t}}{\Gamma\left(\alpha t\right)} x^{\alpha t - 1} e^{-\beta x}, \; x \geq 0, \; \alpha > 0, \beta > 0.$$

F_t and $\bar{F}_t$ cumulative distribution and survival function of X_t.

- The system fails when its degradation exceeds the level L,

$$\sigma_L = \inf(t > 0 \colon X_t > L).$$

- At time σ_L, a signal is sent to the maintenance team which arrives at time $\sigma_L + \tau$ and replaces the system by a new one.
- The system is unavailable from σ_L up to $\sigma_L + \tau$.

Maintenance strategy

Preventive maintenance strategy

- Signal sent to the maintenance team when the system degradation reaches M $\big(0<M<L\big)$ $\big($at time $\sigma_M\big)$.
- At $\sigma_M + \tau$, the maintenance actions start
  - If the system is failed, a **corrective replacement** is performed
  - If $\sigma_M + \tau < \sigma_L$, a **preventive imperfect repair** is performed. After repair
    - If system degradation greater M, **preventive replacement**
    - If system degradation less M, goes on working

Maintenance actions

- Corrective replacement (CR): system is broken at team maintenance arrival
- Preventive repair (PM): repair brings the deterioration below M
- Preventive repair plus a preventive replacement (PM+PR): repair does not bring deterioration below M

First Model

Preventive repair (imperfect)

- First model: Repair removes a part ($\rho\%$) of the degradation accumulated from the last maintenance action ($0 \leq \rho \leq 1$)
- Second model: Repair removes a part ($\rho\%$) of the age accumulated from the last maintenance action ($0 \leq \rho \leq 1$)

Goals

- Derive Markov renewal type equations for some transient measures
  - Transient Reliability
  - Transient Availability
  - Transient Expected Cost
- Comparison the two models of repair

First Model. Markov Renewal Process

Maintained system evolution

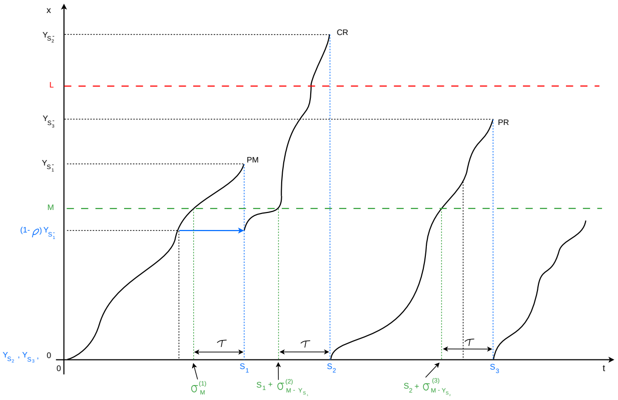
$S_1 = U_1 = \sigma_M^{(1)} + \tau$ 1st maintenance action time, Y_t maintained system evolution

- If $X_{S_1}^{(1)} > L$ CR at S_1, $Y_{S_1} = 0$
- If $X_{S_1}^{(1)} \leq L$ preventive repair (PM) at S_1, reduction of the $\rho\%$ of the degradation
  - $(1-\rho)X_{S_1} \geq M$, unmaintainable system $Y_{S_1} = 0$
  - $(1-\rho)X_{S_1} < M$, $Y_{S_1} = (1-\rho)X_{S_1}$

From Y_{S_1}, 2nd maintenance action is planned at
$S_2 = S_1 + \sigma_{M-Y_{S_1}}^{(2)} + \tau = S_1 + U_2$

- If $Y_{S_2}^{-} > L$ CR at S_2, $Y_{S_2} = 0$
- If $X_{S_2}^{-} \leq L$ PM at S_2, reduction of the $\rho\%$ of the degradation accumulated in U_2
  - $Y_{S_1} + (1-\rho)X_{U_2}^{(2)} \geq M$, unmaintainable system $Y_{S_2} = 0$
  - $Y_{S_1} + (1-\rho)X_{U_2}^{(2)} < M$, $Y_{S_2} = Y_{S_1} + (1-\rho)X_{U_1}^{(2)}$

First Model. Markov Renewal Process



After S_n, evolution of $(Y_t)_{t>S_n}$ depends on $(Y_t)_{t\leq S_n}$ only through Y_{S_n}, (Y_t) semi-regenerative process with underlying MRP (S_n, Y_n) where $Y_n = Y_{S_n}$ and interarrival times $U_n's$

First Model. Markov Renewal Process

$$(S_1, Y_{S_1}^-) = (\sigma_M + \tau, X_{\sigma_M + \tau}) \overset{\mathcal{L}}{=} (\sigma_M, X_{\sigma_M}) + (\tau, X_\tau^{(1)}).$$

$X_\tau^{(1)}$ independent copy of X_τ

Probability distribution function of (σ_M, X_{σ_M}), Bertoin (1998)

$$f_{(\sigma_M, X_{\sigma_M})}(t, y) = \int_{s=0}^{\infty} \mathbf{1}_{\{M \leq y < M+s\}} f_t(y-s)\mu(ds),$$

$\mu(ds)$ Gamma process Levy measure $\mu(ds) = \alpha e^{-\beta s}/s$

Probability distribution function of $(S_1, Y_{S_1}^-)$

For $s > \tau$ and $x > M$

$$h^M(s, x) = \iint_{\mathbb{R}_+^2} \mathbf{1}_{\{M \leq x-y < M+u\}} f_{s-\tau}(x-y-u) f_\tau(y) \mu(du) dy$$

First Model. Markov Renewal Process

Kernel of (S_n, Y_{S_n}) given $Y_0 = x$

$$S_x = \sigma_{M-x} + \tau, \quad , Y_{S_x} = \begin{cases} 0 & Y_{S_x}^- > L-x \\ 0 & Y_{S_x}^- \leq L-x, (1-\rho) Y_{S_x}^- > M-x \\ x + (1-\rho) Y_{S_x}{}^- & Y_{S_x}^- \leq L-x, (1-\rho) Y_{S_x}^- \leq M-x \end{cases}$$

$$q_x(ds,du) = \mathbb{P}\left(S_1 \in ds, Y_{S_1}^- \in du | Y_0 = x\right)$$

$$\delta_0(du)\left(\int_{L-x}^{\infty} h^{M-x}(s,u)du + \mathbf{1}_{\{M-x<(1-\rho)(L-x)\}} \int_{\frac{M-x}{1-\rho}}^{L-x} h^{M-x}(s,u)du\right) ds$$

$$+ \mathbf{1}_{\{u>x\}} \mathbf{1}_{\{u-x<\min((L-x)(1-\rho), M-x)\}} h^{M-x}\left(s, \frac{u-x}{1-\rho}\right) \frac{du}{1-\rho} ds$$

$\widehat{q}_x(s,u)$ kernel restricted to the operating states $Y_0 = x$

$$\widehat{q}_x(s,u) = \delta_0(du) \int_{\frac{M-x}{1-\rho}}^{L-x} h^{M-x}(s,u)du + \mathbf{1}_{\left\{\frac{u-x}{1-\rho} < \min\left(L-x, \frac{M-x}{1-\rho}\right)\right\}} \frac{1}{1-\rho} h^{M-x}\left(s, \frac{u-x}{1-\rho}\right)$$

Reliability Measures. First Maintenance Model

Transient Reliability for $t \leq \tau$

$R_x(t)$ probability system is working in $[0, t]$, $Y_0 = x \in [0, M]$:

$$R_x(t) = \mathbb{P}_x(T > t) = \mathbb{P}(\sigma_{L-x} > t) = \mathbb{P}(X_t < L - x) = F_t(L - x), \quad t < \tau$$

Transient Reliability for $t \geq \tau$

$$R_x(t) = \mathbb{P}_x(T > t, S_1 > t) + \mathbb{P}_x(T > t, S_1 \leq t), \quad t \geq \tau$$

$$\begin{aligned}
\mathbb{P}_x(T > t, S_1 > t) &= \mathbb{P}(\sigma_{L-x} > t, \sigma_{M-x} + \tau > t) = G_x(t) \\
\mathbb{P}_x(T > t, S_1 \leq t) &= \mathbb{E}\left[\mathbf{1}_{\{S_x \leq t\}} \mathbf{1}_{\left\{Y_{S_x}^- < L-x\right\}} \mathbf{1}_{\left\{(1-\rho) Y_{S_x}^- \geq M-x\right\}} R_0(t - S_x)\right] \\
&+ \mathbb{E}\left[\mathbf{1}_{\{S_x \leq t\}} \mathbf{1}_{\left\{Y_{S_x}^- < L-x\right\}} \mathbf{1}_{\left\{(1-\rho) Y_{S_x}^- < M-x\right\}} R_{x + (1-\rho) Y_{S_x}^-}(t - S_x)\right]
\end{aligned}$$

Reliability Measures. First Maintenance Model

Transient Reliability

For $t > \tau$ and $x \in [0, M]$, transient reliability fulfills

$$R_x(t) = G_x(t) + \int_\tau^t \int_0^M R_y(t-s)\widehat{q}_x(ds, dy)$$

where $\widehat{q}_x(ds, dy)$ sub-semi-Markov kernel of (S_n, Y_{S_n}) given $Y_0 = x$ restricted to the operating states

$$\begin{aligned} G_x(t) &= \mathbb{P}_x(T>t, S_1>t) \\ &= \int_0^{M-x} f_{t-\tau}(y) F_\tau(L-x-y)dy, \end{aligned}$$

Reliability Measures. First Maintenance Model

Transient Availability for $t \leq \tau$

$A_x(t)$ probability system is working at t given $Y_0 = x$ and $t < \tau$

$$A_x(t) = \mathbb{P}_x(Y_t{<}L){=}\mathbb{P}(\sigma_{L-x}{>}t){=}\mathbb{P}(X_t{<}L{-}x){=}F_t(L{-}x), \quad t{<}\tau$$

Transient Availability for $t > \tau$

$$A_x(t){=}\mathbb{P}_x(Y_t{<}L, S_1{>}t){+}\mathbb{P}_x(Y_t{<}L, S_1{\leq}t), \quad t{\geq}\tau$$

$$\begin{aligned}
\mathbb{P}_x(Y_t{<}L, S_1{>}t) &= \mathbb{P}(\sigma_{L-x}{>}t, \sigma_{M-x}{+}\tau{>}t) \\
\mathbb{P}_x(T{>}t, S_1{\leq}t) &= \mathbb{E}\left[\mathbf{1}_{\{S_x \leq t\}}\mathbf{1}_{\left\{Y^-_{S_x} \geq L-x\right\}}A_0(t{-}S_x)\right] \\
&+ \mathbb{E}\left[\mathbf{1}_{\{S_x \leq t\}}\mathbf{1}_{\left\{Y^-_{S_x} < L-x\right\}}\mathbf{1}_{\left\{(1-\rho)Y^-_{S_x} \geq M-x\right\}}A_0(t{-}S_x)\right] \\
&+ \mathbb{E}\left[\mathbf{1}_{\{S_x \leq t\}}\mathbf{1}_{\left\{Y^-_{S_x} < L-x\right\}}\mathbf{1}_{\left\{(1-\rho)Y^-_{S_x} < M-x\right\}}A_{x+(1-\rho)Y^-_{S_x}}(t{-}S_x)\right]
\end{aligned}$$

Reliability Measures. First Maintenance Model

Transient Availability

For $t>\tau$, availability fulfills

$$A_x(t)=G_x(t)+\int_\tau^t\int_0^M A_y(t-s)q_x(ds,dy),$$

with $G_x(t)$

$$\begin{aligned} G_x(t) &= \mathbb{P}_x(Y_t>L, S_1>t) \\ &= \int_0^{M-x} f_{t-\tau}(y)F_\tau(L-x-y)dy, \end{aligned}$$

Transient Expected Cost. First Maintenance Model

Transient cost

$c_x(t)$ mean cost in $]0, t]$ given $Y_0 = x$, $x \in [0, M]$

$$c_x(t) = \mathbb{E}_x[C(]0,t])].$$

c_{CR} corrective replacement cost, c_{PR} preventive replacement cost, c_{PM} preventive repair cost and c_d downtime cost per unit time

Transient cost

For $t \leq \tau$

$$c_x(t) \quad = \quad c_d \int_0^t \mathbb{P}(t-u > \sigma_{L-x})\ du = c_d \int_0^t \bar{F}_{t-u}(L-x)\ du,$$

For $t > \tau$

$$c_x(t) = \mathbb{E}_x\left[C(]0,t])\mathbf{1}_{\{S_1 > t\}}\right] + \mathbb{E}_x\left[C(]0,t])\mathbf{1}_{\{S_1 \leq t\}}\right].$$

Transient Expected Cost. First Maintenance Model

Transient cost for $t > \tau$

The expected cost function at time $t{>}\tau$ with $Y_0{=}x$ fulfills

$$c_x(t){=}B_x(t){+}\int_\tau^t\int_0^M c_y(t{-}s)q_x(ds,dy),$$

with $x{\in}[0,M]$, where

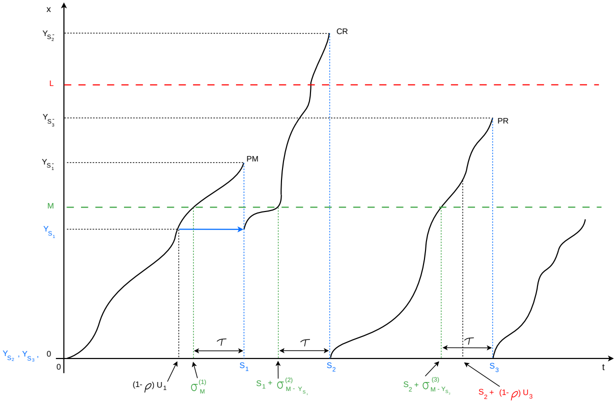
$$\begin{aligned}
B_x(t) &= \mathbb{E}\left[C(]0,t])\mathbf{1}_{\{S_1>t\}}\right]+c_d\mathbb{E}_x\left[(S_1-\sigma_L)^+\mathbf{1}_{\{S_1\le t\}}\right]\\
&+ c_{CR}\mathbb{P}_x\left(S_1\le t, Y_{S_1^-}>L\right)+(c_{PR+c_{PM}})\mathbb{E}_x\left[1_{\{S_1\le t\}}1_{\left\{Y_{S_1}^-\le L\right\}}\mathbf{1}_{\left\{Y_{S_1}>M\right\}}\right]\\
&+ c_{PM}\mathbb{E}_x\left[1_{\{S_1\le t\}}1_{\left\{Y_{S_1}^-\le L\right\}}\mathbf{1}_{\left\{Y_{S_1}\le M\right\}}\right]
\end{aligned}$$

Second Maintenance model

Description (Mercier anc Castro, 2013)

- The system is working. It failed when degradation exceeds level L
- A signal is sent to the maintenance team when degradation reaches level M $(0 < M < L)$ (at time σ_M).
- At $\sigma_M + \tau$, maintenance actions start
  - System is failed at maintenance time, **corrective replacement**
  - System is not failed at $\sigma_M + \tau (< \sigma_L)$, **instantaneous imperfect repair** that removes only some part (ρ%) of the age accumulated from the last maintenance time. After repair
    - Degradation greater M, **preventive replacement**
    - Degradation less M, goes on working

Second Maintenance Model



After S_n, evolution of $(Y_t)_{t>S_n}$ depends on $(Y_t)_{t\leq S_n}$ only through Y_{S_n}, (Y_t) semi-regenerative process with underlying MRP (S_n, Y_n) where $Y_n = Y_{S_n}$ and interarrival times $U_n' s$

Second Model. Markov Renewal Process

Kernel of the Markov Renewal Process $\left(S_n, Y_{S_n}, Y_{S_n^-}\right)$

$$q_x(ds,du,dv)=\mathbb{P}\left(S_1\in ds, Y_{S_1}\in du, Y_{S_1^-}\in dv \,|\, Y_0=x\right), \quad x\in[0,M].$$

$S_1 = \sigma_M + \tau$

$$\left(S_1, X_{(1-\rho)S_1}, X_{S_1}\right)\overset{\mathcal{L}}{=}(\sigma_M, X_{(1-\rho)\sigma_M}, X_{\sigma_M})+(X_\tau, X_{(1-\rho)\tau}, X_\tau\right),$$

Calculating the p.d.f of $(\sigma_M, X_{(1-\rho)\sigma_M}, X_{\sigma_M})$ and p.d.f of $(\tau, X_{(1-\rho)\tau}, X_\tau)$ (τ deterministic), by convolution we get the p.d.f $(S_1, X_{(1-\rho)S_1}, X_{S_1})$ and the kernel

Mistake

But $X_{(1-\rho)\tau} = X_{(1-\rho)(\sigma_M+\tau)} - X_{(1-\rho)\sigma_M}$ is not independent of σ_M

Second Model. Markov Renewal Process

Probability distribution function of $(S_1, X_{(1-\rho)S_1}, X_{S_1})$

Let φ be any measurable function, we compute

$$\mathbb{E}\Big[\varphi(S_1,X_{(1-\rho)S_1},X_{S_1})\Big]=I_1(\varphi)+I_2(\varphi),$$

$$\begin{aligned} I_1(\varphi) &= \mathbb{E}\Big[\varphi(S_1,X_{(1-\rho)S_1},X_{S_1})\mathbf{1}_{\{(1-\rho)(\sigma_M+\tau)>\sigma_M\}}\Big] \\ I_2(\varphi) &= \mathbb{E}\Big[\varphi(S_1,X_{(1-\rho)S_1},X_{S_1})\mathbf{1}_{\{(1-\rho)(\sigma_M+\tau)\leq\sigma_M\}}\Big] \end{aligned}$$

we get

$$u^M(s,u,v)=f_{\rho s}(v-u)\int_0^M f_{s-\tau}(x)\Big(\int_{M-x}^{\infty} f_{\tau-\rho s}(u-t-x)\mu(dt)\Big)dx,\ \tau<s<\tau/\rho,\ M<u<v$$

$$u^M(s,u,v)=f_{(1-\rho)s}(u)\int_M^{\infty} f_{\tau}(v-w)\Big(\int_{w-M}^{\infty} f_{\rho s-\tau}(w-u-t)\mu(dt)\Big)dx,\ s>\tau/\rho,\ u<M<v$$

Second Model. Markov Renewal Process

Kernel of (S_n, Y_{S_n})

The kernel $(\bar{q}_x(ds,dy))$ of (S_n,Y_{S_n})

$$\bar{q}_x(ds,dy)=\nu_x(ds,dy)+\delta_0(dy)\int_{L-x}^{+\infty}\int_0^z u^{M-x}(s,w,z)\ dw\ \ dz$$

for $s>\tau$, $x\in[0,M]$ where

$$\begin{aligned}\nu_x(ds,dy) &= \mathbf{1}_{\{y\leq M\}}\int_{M-x}^{L-x} u^{M-x}(s,y-x,v)\ dv\ \ dy \\ &+ \delta_0(dy)\int_{M-x}^{L-x} dz\int_{M-x}^{z} u^{M-x}(s,y,z)\ dy\end{aligned}$$

Reliability Measures. Second Maintenance Model

Transient Reliability

Transient reliability fulfills

$$
\begin{aligned}
R_x(t) &= F_t(L-x), \quad t<\tau \\
R_x(t) &= G_x(t)+\int_{\tau}^{t}\int_{0}^{M} R_y(t-s)\nu_x(ds,dy), \quad t>\tau
\end{aligned}
$$

where $\nu_x\,(ds, dy)$ sub-semi-Markov kernel (S_n, Y_{S_n}) given $Y_0 = x$ restricted to the operating states

$$
\begin{aligned}
G_x(t) &= \mathbb{P}_x(T>t, S_1>t) \\
&= \int_0^{M-x} f_{t-\tau}(y)F_\tau(L-x-y)dy,
\end{aligned}
$$

Reliability Measures. Second Maintenance Model

Transient Availability

Transient availability fulfills

$$
\begin{aligned}
A_x(t) &= F_t(L-x), \quad t<\tau \\
A_x(t) &= G_x(t)+\int_\tau^t \int_0^M A_y(t-s)\bar{q}_x(ds,dy), \quad t\geq\tau
\end{aligned}
$$

Transient expected cost

$$
\begin{aligned}
c_x(t) &= c_d \int_0^t \bar{F}_{t-u}(L-x)dx, \quad t<\tau \\
c_x(t) &= B_x(t)+\int_\tau^t \int_0^M c_y(t-s)\bar{q}_x(ds,dy), \quad t\geq\tau
\end{aligned}
$$

Numerical examples

Data set

Gamma process parameters $\alpha = 1.5$, $\beta = 3$. Failure threshold $L = 10$, $\tau = 10$, $\rho = 0.5$, $C_c = 100$, $C_r = 5$, $C_p = 60$ and $C_u = 2$ (m.u.). MC simulation, 100 values from 0 to 10, and 40000 realizations in each point

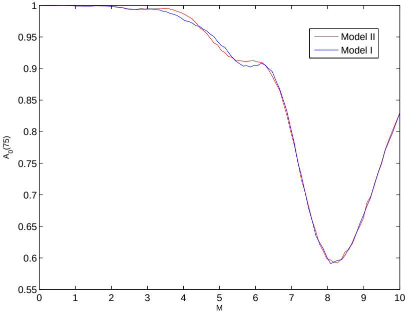


Figure : Availability versus M at time $t = 75$

Numerical examples

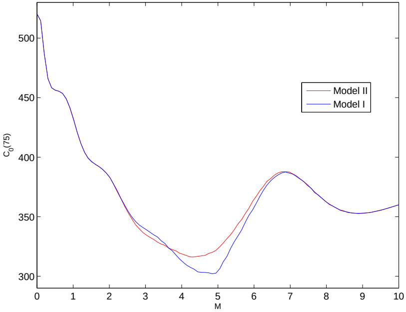


Figure : Expected cost versus M at time $t = 75$

Interpretation

Similar values availability, maximum difference of 19.1295 m.u for the expected cost rate. Model I: 7.69% repairs, 56.96% corrective replacements and 35.35% preventive replacements. Model II, 5.69% repairs, 56.93% corrective replacements and 35.43% preventive replacements

Numerical examples

Data set

$\alpha = 1.5$, $\beta = 3$. Failure threshold $L = 10$, $\tau = 10$, $\rho = 0.75$, $C_c = 100$, $C_r = 5$, $C_p = 60$ and $C_u = 2$ (m.u). MC simulation, 100 values from 0 to 10, and 40000 realizations in each point

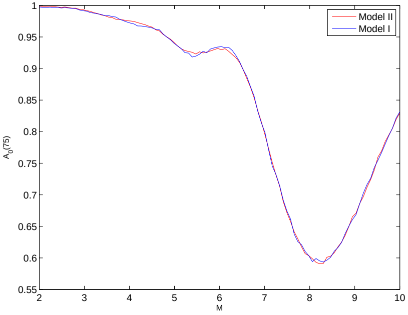


Figure : Availability versus M at time $t = 75$

Numerical examples

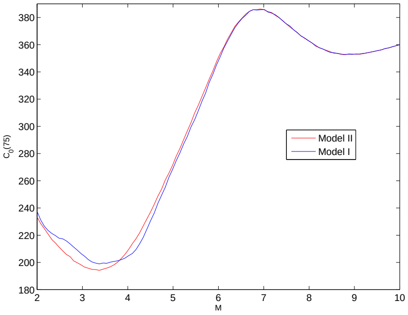


Figure : Expected cost versus M at time $t = 75$

Interpretation

Similar values availability, maximum difference of 10.14 m.u for the expected cost rate. Model I: 30.74% repairs, 55.34% corrective replacements and 13.91% preventive replacements. Model II, 30.68% repairs, 55.42% corrective replacements and 13.90% preventive replacements

Numerical examples

Data set

$\alpha = 1.5$, $\beta = 3$. Failure threshold $L = 10$, $\tau = 2$, $\rho = 0.75$, $C_c = 100$, $C_r = 5$, $C_p = 60$ and $C_u = 2$ (m.u)

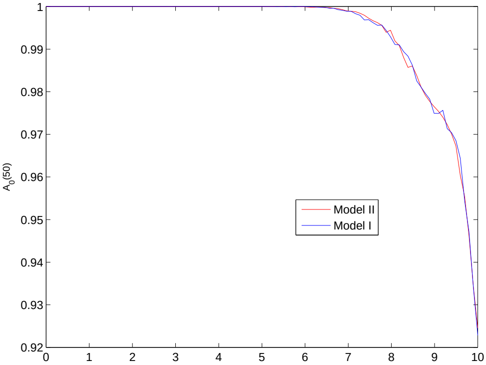


Figure : Availability versus M at time $t = 50$

Numerical examples

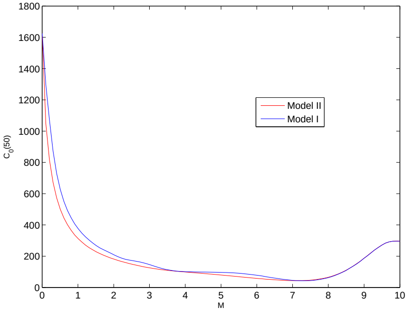


Figure : Expected cost versus M at time $t = 50$

Interpretation

Similar values availability, maximum difference (Model I-Model II) of 255.7646 m.u for the expected cost rate but for low values of $M = 0.20$. Model I: 74.63% repairs, 4.64% corrective replacements and 20.73% preventive replacements. Model II, 69.94% repairs, 4.60% corrective replacements and 25.45% preventive replacements

Numerical examples

Data set

$\alpha = 1.5$, $\beta = 3$. Failure threshold $L = 10$, $\tau = 5$, $\rho = 0.75$

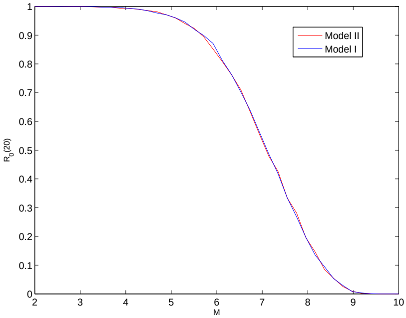


Figure : Reliability versus M at time $t = 20$

Numerical examples

Data set

$\alpha = 1.5$, $\beta = 3$. Failure threshold $L = 10$, $\tau = 3$, $\rho = 0.75$

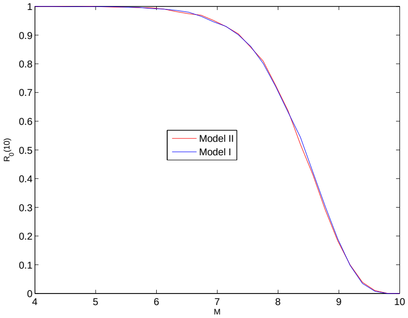


Figure : Reliability versus M at time $t = 10$

Conclusions and further extensions

Conclusions

- Reliability analysis of a continuous degradation modelled as a gamma process with imperfect delay repair under two models considering the overshoot of the gamma process
- Fuctioning of the system is described through a semi-regenerative process
- Some transient reliability measures fulfill Markov renewal equations
- Numerical examples based on **Monte-Carlo simulations** are given. We get that transient and reliability are similar for the two models and the differences between them are found in the expected cost.

Conclusions and further extensions

Further extension: Computing the recursive formulas

For transient measures, Markov renewal equation verifies

$$\begin{aligned} M_x(t) &= W_x(t), \quad t \leq \tau \\ M_x(t) &= H_x(t) + \int_\tau^t \int_0^M M_y(t-s) Q_x(s,y) ds\, dy \quad t > \tau \end{aligned}$$

recursively for $t \leq \tau$

$$M_x(t) = M_x^{(1)}(t) = W_x(t), \quad t < \tau$$

for $(i-1)\tau < t < i\tau$

$$\begin{aligned} M_x(t) &= M_x^{(i)}(t) \\ &= H_x(t) + \sum_{k=1}^{i-1} \int_{t-(k+1)\tau}^{t-k\tau} \int_0^M M_y^{(k)}(t-\tau-w) Q_x(w+\tau,y) dw\, dy, \end{aligned}$$

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